Possible Test of the Vector Nature of Strong Interactions

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A vector theory of the interaction between baryon currents implies the presence of soft-meson radiation in high-energy processes. It is shown that perturbation theory can be used to calculate branching ratios for such radiation in the center-of-mass system. It is also shown that soft mesons are emitted independently of each other. Formulas are derived for the branching ratios. It is suggested that these formulas could be used to determine the ω-baryon coupling constant, if the ω field is the field that interacts with the baryon current.

1. INTRODUCTION

In the last few years gauge theories have been proposed for strong interactions.[1–4] According to those theories the strong interactions are basically vector interactions. Our experimental knowledge provides some simple arguments for the vector character of strong interactions. For instance, the strongly repulsive core of the nucleon-nucleon potential and the strongly attractive forces in nucleon-antinucleon annihilation suggest an analogy with the Coulomb potential in electrodynamics, with the baryon charge playing a role similar to that of the electric charge. A detailed discussion of qualitative arguments for the vector nature of strong interactions was given by Sakurai.[1]

The aim of this paper is to propose a quantitative method for testing the vector nature of the interaction between baryon currents. We restrict ourselves to this interaction which seems to be the strongest of the strong interactions, but our method should easily be applicable to interactions between hypercharge currents.

The idea is the following. Due to the vector nature of electromagnetic interaction, bremsstrahlung of soft photons is present whenever charged particles are accelerated. Similarly, if the interaction between baryon currents is of the vector type soft-meson radiation should occur when baryons are accelerated, i.e., in nonforward scattering or annihilation at high energies.

The analogy with electrodynamics carries us one step further. In the soft-photon problem one can sum the perturbation expansion to all orders, neglecting only terms that vanish with vanishing recoil.[5,6] Thus, the perturbation series is an expansion in the soft-particle recoil rather than in the strength of the interaction. For this reason, we can apply perturbation theory to soft-meson radiation although the interaction is strong.

The fact that the meson mass is nonzero simplifies the problem as compared to electrodynamics. There are no infrared divergences present and there is no need of studying a collective effect because in experiments the mesons can be observed individually. Since, furthermore, the mesons are emitted independently of each other, the problem can, without loss of generality, be reduced to the problem of emission of a single soft meson.

Our method, if it turns out to be experimentally useful, does not only test the vector character of the interaction between baryon currents but it also provides a means to measure its strength.

Among the observed vector mesons there is one candidate for the quantum of the field that mediates the interaction between baryon currents. This candidate is the ω meson, and in the following, we shall, for simplicity, assume that the ω meson is directly coupled to the baryon current.

In order to be able to consider the ω mesons as soft and giving a negligible recoil we shall restrict ourselves to energies and momentum transfers that are large compared to the mass of the ω meson.

Under the assumptions we have made it can be shown that soft ω mesons are emitted independently of each other and that the emission is taken into account by a factor in the transition amplitude. This is the subject of Sec. 2.

In Sec. 3 formulas are given for the branching ratios between processes with and without soft ω emission. The renormalized ω-baryon coupling constant could be determined from measurements of such branching ratios.

Section 4 deals with the case of extremely high energies, when the ω mesons are extremely relativistic but still may be considered as soft. Then the ω mass may be neglected and one has a complete analogy with electrodynamics.

2. MECHANISM OF SOFT MESON RADIATION

It has been shown that at high energies soft meson radiation does not occur if the meson coupling is pseudoscalar.[7]

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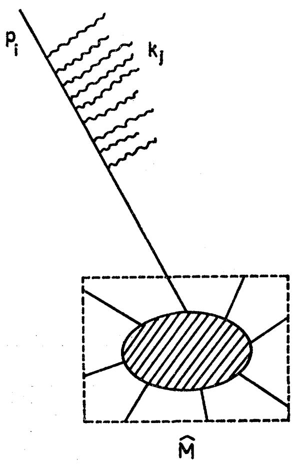

FIG. 1. Diagram for emission of soft mesons from an external baryon line.

We shall see how the mechanism of soft-meson radiation works in a theory with vector ($J_\mu\phi^\mu$) coupling. We can take over almost everything from the perturbation treatment of soft-photon radiation. Everything that tends to zero with the meson recoil will be neglected.

Then the diagrams to be considered are those that are topologically equivalent to infrared divergent diagrams in electrodynamics. Following Nakanishi[8] we have that the order of soft-meson momenta κ is given by

$$\kappa=-D=4\nu-2n-l\,, \tag{1}$$

where ν= the number of independent integration variables k_j (soft-particle momenta); n= the number of boson propagators containing only k_j's; l= the number of propagators with denominators of the type

$$(p+\sum c_jk_j)^2+m^2\,,\quad \text{where}\quad p^2+m^2=0\,. \tag{2}$$

(An external line is to be considered as half a propagator.) From this it follows that

$$\kappa\geqslant 0 \tag{3}$$

in the theory we are considering, and that for $\kappa=0$—which is the case we are interested in—the soft mesons are attached to external or almost external lines [with propagator denominators of the type (2)].

Let M be the invariant transition amplitude for a process $|i\rangle\rightarrow|f\rangle$ in which r baryons (and antibaryons) with momenta $p_1,\cdots,p_r$ and baryonic charges $q_1,\cdots,q_r$ participate. Then

$$\langle f|S-1|i\rangle=\delta(P_f-P_i)M\,, \tag{4}$$

where S is the scattering operator, P_i is the total initial momentum and P_f is the total final momentum. Let us define $\epsilon_1,\cdots,\epsilon_r$ such that $\epsilon_i=1$ (-1) if the ith particle is outgoing (incoming).

Now consider the case that along with our "basic" process $|i\rangle\rightarrow|f\rangle$ m soft ω-mesons are emitted with momenta $k_1,\cdots,k_m$ and polarization vectors $e_1,\cdots,e_m$. In order that the recoil be negligible, we must have in the center-of-mass system

$$\sum_{i=1}^{m}k_{i0}\ll q\,, \tag{5}$$

where q is a typical energy in the basic process. (We shall take q to be the momentum transfer and assume that all momenta p_i are of the same order of magnitude as q. This is true for annihilation and for nonforward scattering at high energies in the c.m. system.) The transition amplitude for the emission process just described will be denoted by

$$M(\mathbf{k}_1,e_1;\cdots;\mathbf{k}_m,e_m)\,. \tag{6}$$

We have seen already that the soft ω mesons are emitted from external or almost external lines where they appear together with soft virtual mesons. To show that the emitted mesons are independent of each other and of the virtual mesons we proceed in the following way.

Let $\hat{M}$ be the residual part of M that one would obtain if all soft virtual mesons were left out. Then let n soft (real or virtual) ω mesons with momenta k_j be coupled to the ith external baryon line of M (Fig. 1). We obtain from perturbation theory the following expression, associated with Fig. 1 if meson factors are left out [$\bar{u}(p_i)\hat{M}_i=\hat{M}$].

$$(gq_i)^n\sum_{\text{perm }(j_1,\cdots,j_n)}\bar{u}(p_i)\frac{\gamma^{\mu_{j_1}}(ip_i-m)\gamma^{\mu_{j_2}}(ip_i-m)\cdots\gamma^{\mu_{j_n}}(ip_i-m)}{2\epsilon_ik_{j_1}\cdot p_i[2\epsilon_i(k_{j_1}+k_{j_2})\cdot p_i]\cdots[2\epsilon_i(k_{j_1}+k_{j_2}+\cdots+k_{j_n})\cdot p_i]}\hat{M}_i\,. \tag{7}$$

Here g is the renormalized ω-baryon coupling constant. Using the anticommutation rules for the γ matrices and the Dirac equation we can transform (7) into

$$\prod_{j=1}^{n}(igq_ip_i^{\mu_j})\sum_{\text{perm }(j_1,\cdots,j_n)}\frac{1}{[\epsilon_ik_{j_1}\cdot p_i][\epsilon_i(k_{j_1}+k_{j_2})\cdot p_i]\cdots[\epsilon_i(k_{j_1}+k_{j_2}+\cdots+k_{j_n})\cdot p_i]}\hat{M}=\prod_{j=1}^{n}\left(\frac{igq_ip_i^{\mu_j}}{\epsilon_ik_j\cdot p_i}\right)\hat{M}\,. \tag{8}$$

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Equation (8) shows that the ω mesons are emitted independently of each other by the ith external baryon line and that the emission of the jth meson is (except for meson factors that will be included later) connected with a factor

$$igq_ip_i{}^{\mu i}/\epsilon_ik_j\cdot p_i. \tag{9}$$

Adding the possibilities for emission from any of the external baryon lines we get the emission factor for one soft ω meson

$$(2\pi)^{3/2}s^{\mu j}(k_j)\,, \tag{10}$$

where

$$s^\mu(k)=\frac{ig}{(2\pi)^{3/2}}\sum_{i=1}^{r}\frac{\epsilon_iq_ip_i{}^\mu}{k\cdot p_i}. \tag{11}$$

When meson factors $[i(2\pi)^{-3/2}e_\mu(2k_0)^{-1/2}$ for a real and $i(2\pi)^{-4}(k^2+m_\omega{}^2)^{-1}$ for a virtual meson$]$ are included, we get for a real soft ω meson characterized by k_j, e_j the emission factor

$$i(2k_{j0})^{-1/2}e_j\cdot s(k_j)\,, \tag{12}$$

whereas each virtual soft meson gives the constant factor

$$-\frac{1}{2\pi i}\int_{\text{soft region}}\frac{d^4k}{k^2+m_\omega{}^2}\left(g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\omega{}^2}\right)s^\mu(k)s^\nu(-k). \tag{13}$$

The total effect of all possible virtual mesons is only a factor that restores $\hat{M}$ into M.

We are now ready to write down the expression for the transition amplitude (6) for emission of m soft ω mesons:

$$M(\mathbf{k}_1,e_1;\cdots;\mathbf{k}_m,e_m)=\prod_{j=1}^{m}(i(2k_{j0})^{-1/2}e_j\cdot s(k_j))M. \tag{14}$$

3. BRANCHING RATIOS

The branching ratio for soft ω-meson emission is now easily obtained from (14) as ("$*$" denotes complex conjugate)

$$B(\mathbf{k}_1,\cdots,\mathbf{k}_m)=\frac{\sum_{\text{pol}}|M(\mathbf{k}_1,e_1;\cdots;\mathbf{k}_m,e_m)|^2}{|M|^2}=\prod_{j=1}^{m}\frac{\sum_{\text{pol}}e_j{}^\mu e_j{}^\nu s_\mu(k_j)s_\nu{}^*(k_j)}{2k_{j0}}. \tag{15}$$

For soft mesons the longitudinal polarization gives a vanishing contribution because the definition (11) of the current $s^\mu(k)$ implies that

$$k_\mu s^\mu(k)=\frac{ig}{(2\pi)^{3/2}}\sum_{i=1}^{r}\epsilon_iq_i=0 \tag{16}$$

because of the conservation of baryonic charge. For this reason, the polarization sum is

$$\sum_{\text{pol}}e^\mu e^\nu s_\mu(k)s_\nu{}^*(k)=\left(g^{\mu\nu}+\frac{k^\mu k^\nu}{m_\omega{}^2}\right)s_\mu(k)s_\nu{}^*(k)=s(k)\cdot s^*(k) \tag{17}$$

and the branching ratio (15) reduces to

$$B(\mathbf{k}_1,\cdots,\mathbf{k}_m)=\prod_{j=1}^{m}B(\mathbf{k}_j)\,, \tag{18}$$

where

$$B(\mathbf{k})=s(k)\cdot s^*(k)/2k_0,\quad (k_0=(\mathbf{k}^2+m_\omega{}^2)^{1/2}). \tag{19}$$

Inserting (11) into (19) we find that

$$B(\mathbf{k})=\left(\frac{g}{2\pi}\right)^2\frac{1}{4\pi}\sum_{i,j=1}^{r}\frac{\epsilon_i\epsilon_jq_iq_jp_i\cdot p_j}{k_0(k\cdot p_i)(k\cdot p_j)},\quad (k_0=(\mathbf{k}^2+m_\omega{}^2)^{1/2}). \tag{20}$$

This is the factor which one soft meson of momentum $\mathbf{k}$ contributes to the branching ratio (18).

Integrating over the directions of an emitted meson,[9] we obtain the energy spectrum

$$\begin{aligned}B(k_0)&=\int_{(k_0=(\mathbf{k}^2+m_\omega{}^2)^{1/2})}d\Omega_{\mathbf{k}}|\mathbf{k}|k_0B(\mathbf{k})\\&=\left(\frac{g}{2\pi}\right)^2\frac{1}{(k_0{}^2-m_\omega{}^2)^{1/2}}\left[-\sum_{i=1}^{r}\frac{q_i{}^2p_i{}^2}{p_i'^2}-\sum_{1\leqslant i<j\leqslant r}\frac{\epsilon_i\epsilon_jq_iq_jp_i\cdot p_j}{p_i'\cdot p_j'C_{ij}}\ln\frac{1+C_{ij}}{1-C_{ij}}\right],\end{aligned} \tag{21}$$

with

$$C_{ij}=\left[1-\frac{p_i'^2\cdot p_j'^2}{(p_i'\cdot p_j')^2}\right]^{1/2},\qquad p_i'=(ap_{i0},\mathbf{p}_i)\,,\quad a=(1-(m_\omega{}^2/k_0{}^2))^{-1/2}. \tag{22}$$

We now see that measurements of $B(\mathbf{k})$ or $B(k_0)$ could provide information on the ω-baryon coupling constant g.

4. EXTREME HIGH-ENERGY CASE

In the extreme high-energy case the ω mass is negligible. Then we can set $a=1$ and $p_i'=p_i$ in (22) and (21) is reduced to an expression similar to C in Eq. (54) of Ref. 6. The collective energy loss through ω radiation can be dealt with just like in Ref. 6. This extreme case is, of course, not yet of any experimental interest.

[9] For the integration it is most convenient to use a parametrization method as for instance in Ref. 6, p. 1022.

5. CONCLUSIONS

The vector-meson theory of strong interactions could be tested through measurements of the branching ratios for soft (in the c.m. system) meson radiation in non-forward meson or nucleon scattering at high energies or in antinucleon annihilation at high energies.

Under the assumption that the ω meson is connected with the field that mediates the interactions between baryon currents Eq. (21) can then be used to determine the coupling constant g. If the value obtained in this way is reasonable ($\gtrsim$ a few units) one may consider the experiment as a support for the vector meson theory of strong interactions and for the assumption that the ω field is coupled to the baryon current. The value of g can then be considered as the measured value of the renormalized ω-baryon coupling constant.

If the value obtained for g is unreasonably small this could be taken as an evidence against the vector meson theory of strong interactions.

However, it is by no means certain that the relations between fundamental fields and physical particles are so simple as assumed here, but they may be of a complicated and remote character.[3] For this reason the experiments suggested here can only give indications concerning the nature of strong interactions but cannot serve as a firm basis for conclusions.

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Remarks Concerning Possible Higher Resonances in the Unitary Symmetry Model*

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A resonance which "decays" into a meson and one of the $J=\frac{3}{2}^+$ isobars may, according to the unitary symmetry scheme, belong to any one of the irreducible representations **8**, **10**, **27**, or **35**. Even if only a $Y=1$ member of the supermultiplet is detected, it is still possible to determine the dimensionality of the representation by the study of the ratios between the partial widths for decays into different isobar-meson channels.

There appears to be a growing amount of evidence to support the identification of unitary symmetry—the invariance under SU_3—as the higher symmetry which underlies charge independence in strong interaction physics.[1] The existing stable particles, the vector mesons, and the low-lying baryon-meson resonances which have been discovered in the past few years may be classified according to the irreducible representations of this group, and it appears that the lowest states of the particles with baryon number $B=1$ belong to the representations **8** and **10**, while the lowest states with $B=0$, the pseudoscalar and vector mesons, belong to **8**. There also appears to be a vector meson belonging to the **1**. It is of some interest to speculate about the existence of resonances with $B=1$ belonging to higher dimensional irreducible representations of SU_3. In particular, we shall be concerned with resonances which "decay" into a meson and one of the known isobars, N_3^*, Y_1^* or one of its other partners in the **10** representation. Since

$$\mathbf{8}\otimes\mathbf{10}=\mathbf{35}\oplus\mathbf{27}\oplus\mathbf{10}\oplus\mathbf{8},$$

it follows that the hypothetical resonance belongs to one of the irreducible representations in the decomposition of this product, and we shall concern ourselves with the identification of the dimensionality of the resonances. In principle, one could just count resonances, but as the isospin multiplets within a unitary supermultiplet are usually split, and as some of the resonances, even if they exist, may be rather hard to produce, it is of some interest to look at some more indirect ways of establishing the dimensionality.

In Table I[2] we list the contents of the relevant super-

Table I. Isospin (T) and hypercharge (Y) for various supermultiplets.

Representation	35	27	10	8
$Y=2$	$T=2$	$T=1$	···	···
$Y=1$	$T=\frac{5}{2},\frac{3}{2}$	$T=\frac{3}{2},\frac{1}{2}$	$T=\frac{3}{2}$	$T=\frac{1}{2}$
$Y=0$	$T=2,1$	$T=2,1,0$	$T=1$	$T=1,0$
$Y=-1$	$T=\frac{3}{2},\frac{1}{2}$	$T=\frac{3}{2},\frac{1}{2}$	$T=\frac{1}{2}$	$T=\frac{1}{2}$
$Y=-2$	$T=1,0$	$T=1$	$T=0$	···
$Y=-3$	$T=\frac{1}{2}$	···	···	···

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[1] See S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963) and references cited therein.

[2] The novice in SU_3 will find some simple methods for obtaining the isospin content of irreducible representations, for reducing out products of irreducible representations, and for calculating the generalized Clebsch-Gordan coefficients which appear in the wave functions in this article, in a report written by the author, Argonne National Laboratory, ANL-6729 (unpublished).